SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2008

HSC Task 1

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Start each **NEW** question in a separate answer booklet.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.

Total Marks – 64

- Attempt questions 1-5.
- All answers are to be given in simplest exact form unless specified.

Examiner: *D.McQuillan*

Start on a separate answers sheet.

Question 1 (13 marks) **Marks**

(a) Evaluate $\log_{10} 12 - \log_{10} 2$ to 3 decimal places. 1

(b) Evaluate $\sum_{n=2}^{5} 3n - 1$. 1

(c) Find using the formula $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ find the distance between the line $5x - 12y + 15 = 0$ and the point $(-1, 3)$. 1

(d) Find the domain and range of the following 6

(i) $y = |2x - 6|$

(ii) $y = \sqrt{9 - x}$

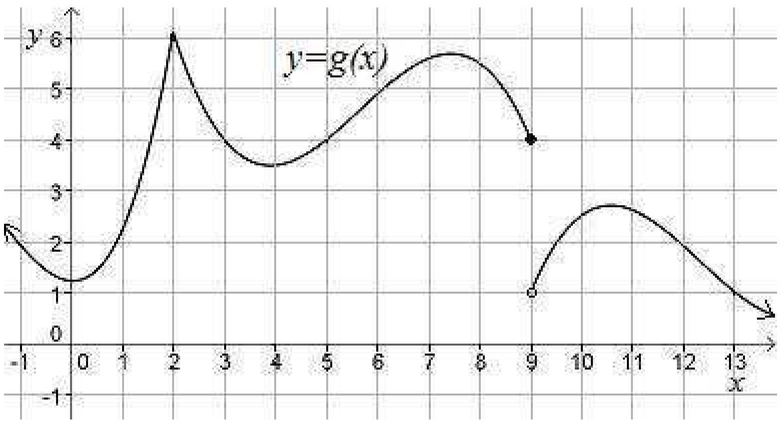
(iii) $y = \dfrac{x}{|x|}$

(e) For the series $2 + 5 + 8 + 11 + \cdots$ 2

(i) Find the value of the 20^{th} term.

(ii) Find the sum of the first 20 terms.

(f) 2



(i) For what x value(s) is $g(x)$ not continuous?

(ii) For what x value(s) is $g(x)$ not differentiable?

End of Question 1

Start on a separate answers sheet.

Question 2 (13 marks) **Marks**

(a) Solve $3^x = 81$ for x. 1

(b) Show that $3x^2 - 4x + 2 = 0$ has no real roots. 1

(c) Express $2x^2 - 5x + 8$ in the form $Ax(x-1) + Bx + C$. 2

(d) Differentiate the following 5

(i) $3x^3 - 9$

(ii) $\dfrac{x^2}{x+1}$

(iii) $x\sqrt{x^2+1}$

(e) For $f(x) = x^2 + 6x - 16$ find, 4

(i) the minimum value of $f(x)$.

(ii) the values of x such that $f(x) \geq 0$.

End of Question 2

Start on a separate answers sheet.

Question 3 (13 marks) **Marks**

(a) Is the function $f(x) = \dfrac{3x^3 + x - 1}{x}$ ODD, EVEN or NEITHER? 1

(b) If $\log_b 3 = 1.09$ and $\log_b 4 = 1.38$ find 2

(i) $\log_b 12$

(ii) $\log_b \dfrac{2}{3}$

(c) If $f(x) = \dfrac{2x^2 + 5x - 3}{4x^2 + 16x + 12}$, find the value of: 4

(i) $\lim\limits_{x \to 0} f(x)$

(ii) $\lim\limits_{x \to -3} f(x)$

(iii) $\lim\limits_{x \to \infty} f(x)$

(d) If α and β are the roots of the equation $2x^2 + 7x + 4 = 0$, find the value of: 4

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$

(e) What is the equation of the parabola with vertex (1, 2) and directrix $y = -1$? 2

End of Question 3

Start on a separate answers sheet.

Question 4 (13 marks) **Marks**

(a) Dean undertook to pay a charity \$200 one year, \$150 the next year, three-quarters of \$150 the third year and so on until he died. 4

(i) What is the greatest sum of money the charity may expect from these donations?

(ii) If Dean died after making 20 donations, by how much, to the nearest dollar, would the total received by the charity fall short of the maximum expectation?

(b) Given that $f(x) = x^2 - 6x$. 3

(i) Find $f'(x)$ using differentiation by first principles.

(ii) Hence show that $2xf'(x) - f(2x) = 0$.

(c) Given the points O, the origin, and $A(0, 2a)$; find and graph the locus of P such that $OP \perp PA$. 3

(d) If $\log_b a = p$ and $c = a^2$, find, in terms of p. 3

(i) $\log_b c$

(ii) $\log_c b$

End of Question 4

Start on a separate answers sheet.

Question 5 (12 marks) **Marks**

(a) Sketch the graph of $y = \log_2(x+2)$ showing all important features. 2

(b) Sketch the following region. 3

$$y \le \sqrt{25 - x^2}$$
$$x + 2y - 5 < 0$$

(c) Each summer 10% of trees on a certain plantation die out, and each winter, workmen plant 100 new trees. At the end of winter 1990 there were 1200 trees in the plantation. 7

(i) How many living trees were there at the end of winter 1980?

(ii) When will the plantation fall below 1100 trees after the winter plantings?

(iii) What will happen to the plantation into the future if conditions remain unchanged?

End of Question 5

End of Exam

Question 1:

a) $\log_{10}12 - \log_{10}2$

$= 0.77815125$

$= 0.778$ (3dp) 1

b) $\sum_{n=2}^{5} 3n - 1$

$= 5 + 8 + 11 + 14$

$= 38$ 1

OR

$S_n = \frac{n}{2}(a + l)$

$= \frac{5}{2}(5 + 14)$

$= 38$

c) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|5 \times -1 + -12 \times 3 + 15|}{\sqrt{5^2 + (-12)^2}}$

$= \frac{|-5 - 36 + 15|}{\sqrt{169}}$

$= \frac{|-26|}{13}$

$= \frac{26}{13}$

$= 2$ 1

d) i)

Domain: all x 1

Range: $y \geqslant 0$ 1

ii)

Domain: $x \leqslant 9$ 1

Range: $y \geqslant 0$ 1

iii)

Domain: $x \neq 0$ 1

Range: $y = -1$ OR 1 1

e) i) $d = 3$ $a = 2$

$T_{20} = a + (n-1)d$

$= 2 + 19 \times 3$

$= 59$ 1

ii) $S_n = \frac{20}{2}(2 + 59)$

$= 610$ 1

OR

$= \frac{20}{2}(2 \times 2 + (19)3)$

$= 610$

f) i) $x = 9$ 1

ii) $x = 2 + 9$ 1

13

QUESTION 2

(a) $3^x = 81$

$\log_3 81 = x$

$x = 4$

(b) $\Delta = 16 - 4\times3\times2$

$= -8$

$\Delta < 0$ ∴ no real roots

(c) $2x^2 - 5x + 8 = 2x^2 - 2x - 3x + 8$

$= 2x(x-1) - 3x + 8$

(d) (i) $9x^2$

(ii) $\dfrac{(x+1)\times 2x - x^2\times 1}{(x+1)^2}$

$= \dfrac{x^2+2x}{(x+1)^2}$

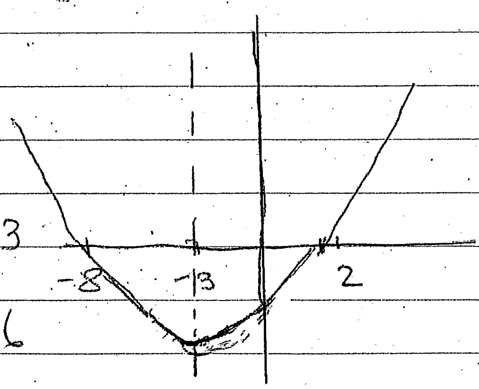
(iii) $\sqrt{x^2+1}\times 1 + x\times\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\times 2x$

$= \sqrt{x^2+1} + \dfrac{x^2}{\sqrt{x^2+1}}$

(e)

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$



(i) axis of symmetry $x = -3$

min value $(-3)^2 + 6(-3) - 16$

$= -25$

(ii) $f(x) \geq 0$

when $x \leq 8$ or $x \geq 2$

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3 (a) $f(x) = \dfrac{3x^3+x-1}{x}$

Is it even? $f(x) = f(-x)$

$$f(-x) = \frac{3(-x)^3+(-x)-1}{(-x)}$$

$$= \frac{-3x^3-x-1}{-x}$$

$$= \frac{3x^3+x+1}{x} \quad \text{NO!}$$

Is it odd? $f(x) = -f(-x)$

$$-f(-x) = \frac{-3x^3-x-1}{x} \quad \text{NO!}$$

$f(x)$ is neither 1

(b) $\log_6 4 = 1.38 \Rightarrow 2\log_6 2 = 1.38$

$\log_6 2 = 0.69$ and $\log_6 3 = 1.09$

(i) $\log_6 12 = \log_6 4 + \log_6 3$

$= 1.38 + 1.09$

$= 2.47$ 1

(ii) $\log_6 2 - \log_6 3$

$= 0.69 - 1.09 = -0.4$ 1

(c) $2x^2+5x-3=(2x-1)(x+3)$

$$4x^2+16x+12 = 4(x^2+4x+3)$$
$$= 4(x+3)(x+1)$$

$$f(x) = \frac{(2x-1)\cancel{(x+3)}}{4\cancel{(x+3)}(x+1)} = \frac{2x-1}{4(x+1)}$$

(i) $\lim\limits_{x\to 0} \quad \frac{-1}{4}$ 1

(ii) $\lim\limits_{x\to -3} \quad \frac{-7}{4\times-2} = \frac{-7}{-8} = \frac{7}{8}$ 1

(iii) $\lim\limits_{x\to \infty} \quad \frac{\cancel{x^2}\left(2+\frac{5}{x}-\frac{3}{x^2}\right)}{\cancel{x^2}\left(4+\frac{16}{x}+\frac{12}{x^2}\right)} \to \frac{1}{2}$ 2

(d) $a=2$
$b=7$
$c=4$

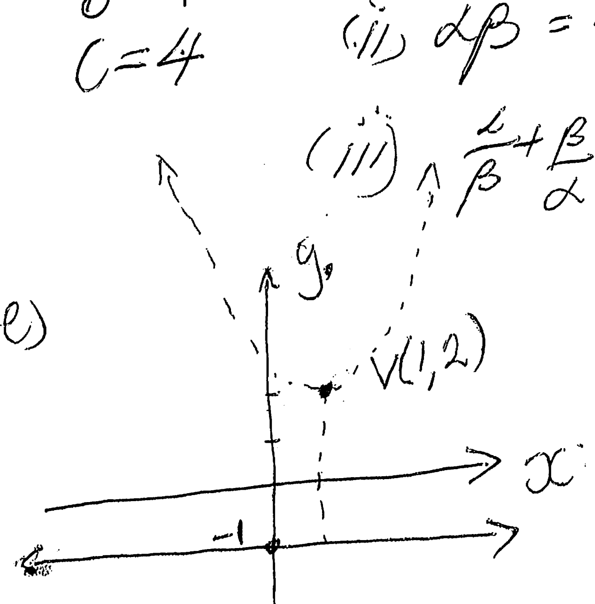
(i) $\alpha+\beta = -\frac{b}{a} = \frac{-7}{2}$ 1

(ii) $\alpha\beta = \frac{c}{a} = \frac{4}{2} = 2$ 1

(iii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{\frac{49}{4}-4}{2}$

$= 4\frac{1}{8}$ 2

(e)



$a=3$

$(x-h)^2=4a(y-k)$

$(x-1)^2=12(y-2)$ 2

Question 4

(a)

(i) $200 + 150 + 112\frac{1}{2} + \ldots$

limiting sum

$$S = \frac{a}{1-r}$$

$$= \frac{200}{1 - 3/4} \quad 1$$

$$= \$800.$$

(ii) $$S_{20} = \frac{200\left[\left(\frac{3}{4}\right)^{20} - 1\right]}{\frac{3}{4} - 1} \quad 2$$

$$= \$797.46$$

$$S - S_{20} = \$2.54 \quad 1$$

b) $f(x) = x^2 - 6x$

2 (i) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^2 - 6(x+h)] - [x^2 - 6x]}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2xh - 6h}{h}$$

$$= \lim_{h \to 0} 2x + h - 6$$

$$= 2x - 6$$

(ii)

1 $$\text{LHS} = 2x f'(x) - f(2x)$$

$$= 2x(2x-6) - (4x^2 - 12x)$$

$$= 4x^2 - 12x - 4x^2 + 12x$$

$$= 0$$

$$= \text{RHS}$$

(c) $$m_{OP} \times m_{PA} = -1$$

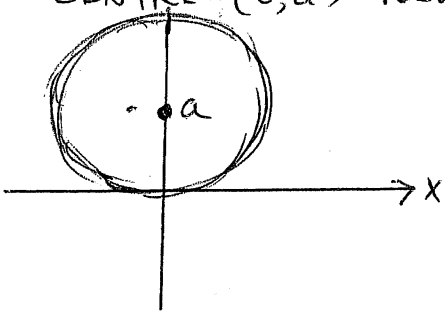
$$\Rightarrow \frac{y}{x} \times \frac{y-2a}{x} = -1 \quad 2$$

$$y^2 - 2ay + x^2 = 0$$

$$x^2 + y^2 - 2ay + a^2 = a^2$$

$$x^2 + (y-a)^2 = a^2$$

CIRCLE CENTRE $(0, a)$ radius a



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(d)

(i) $$\log_b c = \log_b a^2 = 2\log_b a$$

$$= 2p \quad 1$$

(ii) $$\log_c b = \frac{\log_b b}{\log_b c} \quad 2$$

$$= \frac{1}{2p}$$

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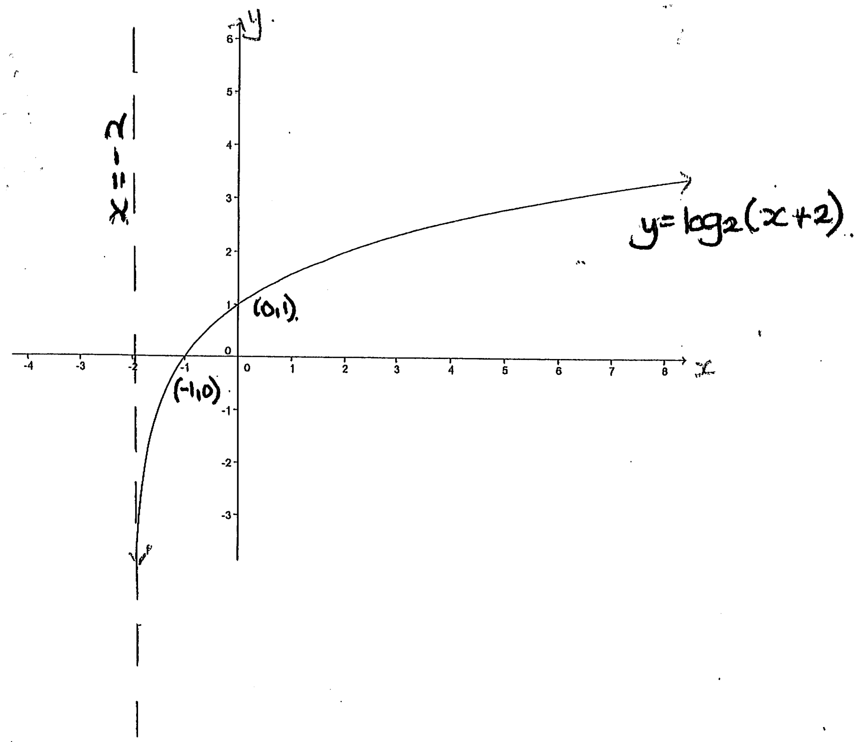
QUESTION FIVE (a)



QUESTION FIVE (b)

$y=\sqrt{25-x^2}$ $\quad$ $x=-2y+5$

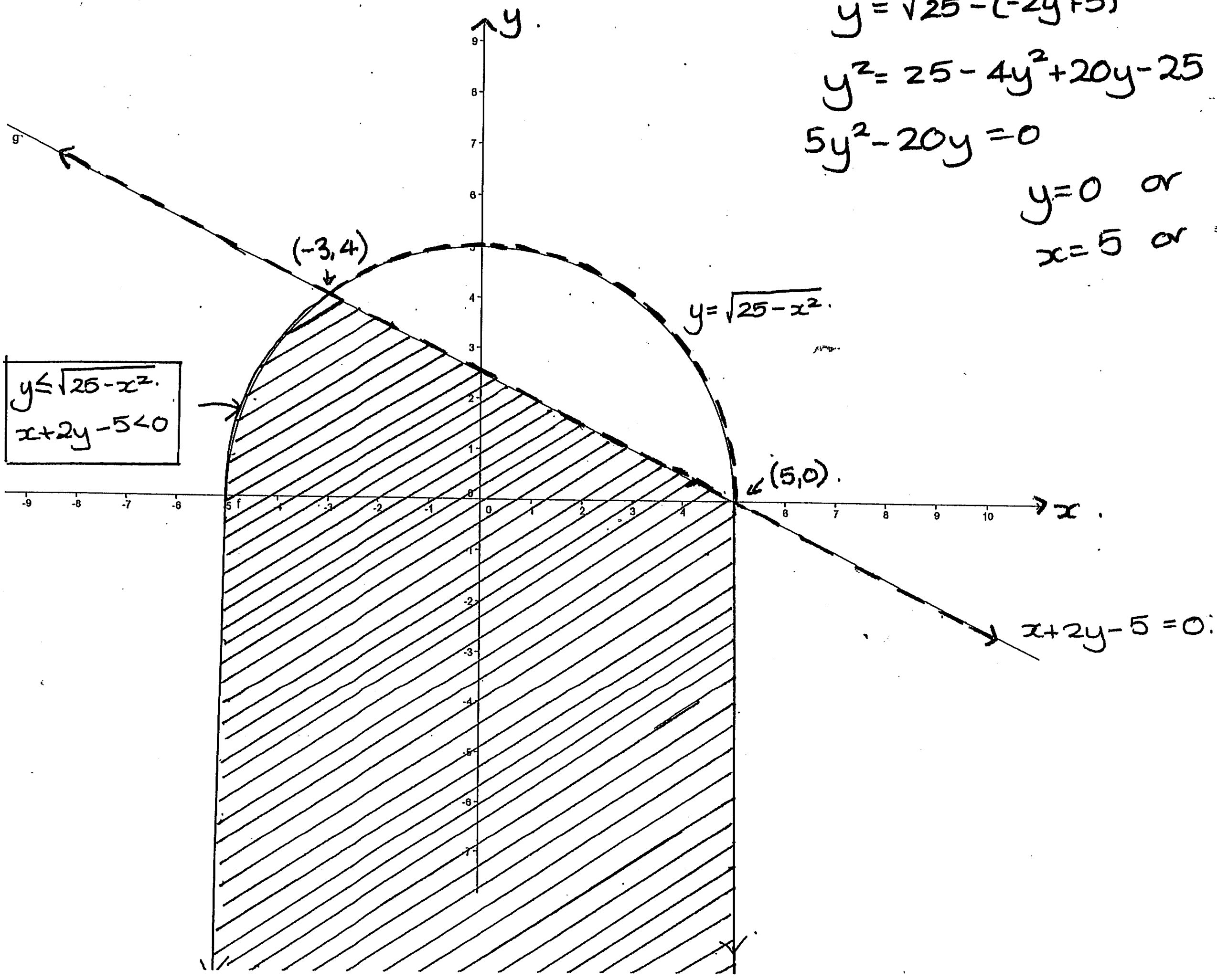
$y=\sqrt{25-(-2y+5)^2}$

$y^2=25-4y^2+20y-25$

$5y^2-20y=0$

$y=0$ or 4

$x=5$ or -3.



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QUESTION 5.

(i). $1990 = A_0 = 1200$ trees.

$1989 = A_1 = (A_0 - 100) + \frac{(A_0 - 100)}{90} \times 10$

$= 10/9\,(A_0 - 100)$

$= 10/9\,(1200 - 100)$

$1988 = A_2 = \frac{10}{9}(A_1 - 100)$

$= 10/9\,[10/9\,(1200 - 100) - 100]$

$= (10/9)^2(1200) - (10/9)^2(100) - (10/9)(100)$

$1987 = A_3 = 10/9\,(A_2 - 100)$

$= 10/9\,[(10/9)^2(1200) - (10/9)^2(100) - (10/9)(100) - 100]$

$= (10/9)^3(1200) - (10/9)^3(100) - (10/9)^2(100) - 10/9\,(100)$

$A_n = (10/9)^n(1200) - (100)[(10/9)^1 + (10/9)^2 + (10/9)^3 + \ldots (10/9)^n]$

* $1980 = A_{10} = (10/9)^{10}(1200) - 100\sum_{r=1}^{n}(10/9)^r$

$= (10/9)^{10}(1200) - 100\left[\frac{(10/9)(\frac{10}{9}^n - 1)}{10/9 - 1}\right]$

$= 3441.5664 - 100(18.6797)$

$\doteq 1573.594398.$

In 1980 there were approx 1574 trees.

(ii) $1990 = A_0 = 1200.$

$A_1 = 1991 = A_0 - (0.1)A_0 + 100$

$= 0.9(A_0) + 100 = 0.9(1200) + 100$

$A_2.\ 1992 = A_1(0.9) + 100$

$= (0.9)[(0.9)(1200) + 100] + 100$

$= (0.9)^2(1200) + (0.9)(100) + 100$

$A_3 = 1993 = A_2(0.9) + 100$

$= (0.9)^3(1200) + (0.9)^2(100) + (0.9)(100) + 100$

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QUESTION 5 (cont)

ii) $A_n = (0.9)^n(1200) + (0.9)^{n-1}(100) + \quad + (0.9)(100) + 100$

$= (0.9)^n(1200) + (100)(1 + 0.9 + \dots 0.9^{n-1})$

$= (0.9)^n(1200) + 100\left(\frac{1(1-0.9^n)}{0.1}\right)$

The plantation will fall below 1100 trees when

$A_n < 1100.$

$0.9^n(1200) + 1000(1 - 0.9^n) < 1100.$
$0.9^n(1200) + 1000 - (0.9)^n(1000) < 1100.$

$200(0.9)^n + 1000 < 1100$
$200(0.9)^n < 100$
$(0.9)^n < 1/2.$

$n(\log 0.9) < \log(0.5)$
$n > \frac{\log(0.5)}{\log(0.9)}$ $\quad \log 0.9 < 1$

$n > 6.5788.$

After 7 years the plantation will fall below 1100 trees ie. after 1997 winter plantings

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QUESTION 5 (iii).

If conditions remain unchanged then the plantation will tend to a limit.

as $n \to \infty$

$$\lim_{n\to\infty}\left[(0.9)^n(1200)+100\left(1+0.9+0.9^2+\ldots+0.9^{n-1}\right)\right]$$

$$=\lim_{n\to\infty}\left[0.9^n(1200)\right]+100\lim_{n\to\infty}\left(1+0.9+0.9^2+\ldots+0.9^{n-1}\right)$$

$$= 0\times 1200 + 100\left[S_\infty\right] \qquad S_\infty = \frac{a}{1-r}$$

$$= 0 + 100\left[\frac{1}{1-0.9}\right]$$

$$= 1000.$$

The number of trees will tend to a limit of 1000 into the future if conditions remain unchanged.